4 Problems for spin chain lecture

You don't have to do all three problems. I think picking one problem that you like will be good enough.

1. Define the spin-1 exchange operator $P_{ij}|\sigma_i, \sigma_j\rangle = |\sigma_j, \sigma_i\rangle$, where $\sigma_{i,j} = +1, 0, -1$ run over the three spin states of $S = 1$. Can you find a matrix representation that does the same thing? You will find that up to some constants and prefactors the answer is $\mathbf{\Lambda}_i \cdot \mathbf{\Lambda}_j$, wehre $\Lambda = (\Lambda_1, \cdots \Lambda_8)$ is a collection of eight, 3×3 matrices called the Gell-Mann matrices. If you know something about the Lie group, you should know that Pauli matrices are generators of the group SU(2), and Gell-Mann matrices those of the group SU(3).

2. Find the Bethe ansatz solution of the $S = 1/2$ Heisenberg spin chain with $N = 4, M = 2$. Pick the one with the lowest energy, which is the ground state of the $N = 6$ antiferromagnetic Heisenberg chain.

3. Can you derive $\langle \mathbf{n}|\dot{\mathbf{n}}\rangle = S(\cos\theta - 1)\dot{\phi}$ for arbitrary spin S? With any spin size the coherent state is always defined by

$$|\mathbf{n}\rangle = e^{-i\phi J_z} e^{-i\theta J_z} \mathbf{n}_0 \tag{31}$$

with $\mathbf{n}_0 = (1, 0, \cdots, 0)^T$.